

# Theoretical estimation of nucleation field in bistable amorphous ferromagnetic microwires

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## Abstract

Correlation between the nucleation field and internal parameters has been derived analytically for amorphous ferromagnetic microwires. Anisotropy distribution specific for amorphous microwires has been fully taken into account instead of being averaged out. Dependence on some anisotropy distribution parameters of the wire was established numerically.

## 1 Introduction

Amorphous ferromagnetic microwires are currently under intensive experimental [1, 2, 3] and theoretical [4, 5] investigation because of their unique properties and possible applications [6, 7]. One of the important directions of study is the case of microwires with positive magnetostriction, which results in bistable behaviour of the wires with an extremely fast domain wall propagation [8]. There are many works on the dependence of velocity on external magnetic field and stress [8], switching field values in various conditions [9], local nucleation field [10] and shape of the moving domain wall [11, 12]. Yet, there are still many unknowns. In particular, the extremely elongated shape of the moving domain wall needs an explanation and thus more theoretical work is required on the propagation process. This problem is also connected to the speed of domain walls, since it is significantly higher than in bulk samples.

The aforementioned elongated shape of the moving domain wall suggests that there are some important processes occurring near the surface of the wire. If one looks at the anisotropy distribution within bistable microwires [4] then one can see that near the surface the anisotropy rapidly changes its magnitude and type. In this work we studied domain wall nucleation taking into account the rapidly changing anisotropy. Analytic calculations gave us connection between nucleation field and material parameters of the wire, while numerical simulations shown dependence of the nucleation field on the geometry of the wire as well as shown where exactly a new domain forms.

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## 2 Methodology

### 2.1 Model

The starting point of our investigation of ferromagnetic microwires is the renown Landau-Lifshitz micromagnetics model [13] with two types of axial anisotropy (both easy axis - one in the axial and the other in the radial direction) and external magnetic field along the axis of the wire. With energy density

$$H = \frac{J}{2} (\nabla \cdot \vec{S})^2 - D_1 (\vec{S} \cdot \hat{z})^2 - D_2 (\vec{S} \cdot \hat{r})^2 - g\mu_B \vec{B} \cdot \vec{S} \quad (1)$$

where  $J$  is the exchange energy density,  $S$  the normalised local magnetisation vector,  $D_1$  and  $D_2$  as respective anisotropy coefficients (in general non-constant),  $g$  as standard gyromagnetic factor,  $\mu_B$  as Bohr magneton and  $B$  as external magnetic field intensity. The equation of motion has form

$$\hbar \frac{\partial \vec{S}}{\partial t} = -\vec{S} \times \nabla_S H + \lambda \vec{S} \times (\vec{S} \times \nabla_S H) \quad (2)$$

where  $\nabla_S$  is a regular vector differential operator over components of  $\vec{S}$  at given points instead of space coordinates and  $\lambda$  is the Landau-Lifshitz damping coefficient.

Considering the estimations of stress inside bistable microwires with glass coating [4] and calculations of anisotropy variation along the radius of the wire presented in the same paper we can divide the wire into three distinct layers

- Surface with very high radial anisotropy.
- Intermediate layer, where the type and strength of anisotropy changes sharply.
- Bulk of the wire, where axial anisotropy is dominating and there is little variation in its strength.

Considering the very high anisotropy near the surface perpendicular to the external magnetic field, we can assume that this layer does not change significantly during the magnetisation switching, so it will serve as a boundary condition for the intermediate layer.

Since in the intermediate layer the anisotropy changes direction and is relatively low, domain wall nucleation is much easier in this region. Therefore we can expect the new domain to originate in the intermediate layer and expand into the bulk of the wire. Such an assumption is in tune with the empirical estimation of moving domain wall shape obtained by Panina et al. [11].

### 2.2 Reduction

Within the intermediate layer we assume the anisotropy coefficients to be linear in radius up to the point  $(r_0)$ , where the anisotropy changes its type and both types of anisotropy change at the same rate  $d$ . The model needs to take into account that the increase of anisotropy toward the centre of the wire stops at some specific radius  $(r_k)$ . In particular, we assume

$$D_1(r) = \frac{d}{4} (-r + 2r_0 - r_k - |r - r_k| + |r - 2r_0 + r_k + |r - r_k||) \quad (3)$$

$$D_2(r) = d \frac{|r - r_0| + r - r_0}{2} \quad (4)$$

When we input this form into equations of motion we obtain

$$\begin{aligned}
\hbar \frac{\partial \vec{S}}{\partial t} = & -\vec{S} \times \left( -J\Delta \vec{S} - \frac{d}{2} (-r + 2r_0 - r_k - |r - r_k| + |r - 2r_0 + r_k + |r - r_k|) \vec{S} \cdot \hat{z} \hat{z} \right. \\
& \left. - d(|r - r_0| + r - r_0) \vec{S} \cdot \hat{r} \hat{r} - g\mu_B \vec{B} \right) \\
& + \lambda \vec{S} \times \left( \vec{S} \times \left( -J\Delta \vec{S} - \frac{d}{2} (-r + 2r_0 - r_k - |r - r_k| + |r - 2r_0 + r_k + |r - r_k|) \vec{S} \cdot \hat{z} \hat{z} \right. \right. \\
& \left. \left. - d(|r - r_0| + r - r_0) \vec{S} \cdot \hat{r} \hat{r} - g\mu_B \vec{B} \right) \right)
\end{aligned} \tag{5}$$

For further numerical analysis it would be beneficial to minimise the number of parameters in the equation. In order to perform the reduction, we can scale the time and space variables as well as multiply the whole equation by a constant. With those procedures we can arrive at an equation containing only four independent parameters two of which ( $r'_k$  and  $r'_0$ ) are connected with geometry of the wire

$$B' = g\mu_B B (d^2 J)^{-\frac{1}{3}} \tag{6}$$

$$\lambda \tag{7}$$

$$r'_k = \left( \frac{d}{J} \right)^{\frac{1}{3}} r_k \tag{8}$$

$$r'_0 = \left( \frac{d}{J} \right)^{\frac{1}{3}} r_0 \tag{9}$$

and a new set of variables

$$t' = \frac{(d^2 J)^{\frac{1}{3}}}{\hbar} t \tag{10}$$

$$\forall_i x'_i = \left( \frac{d}{J} \right)^{\frac{1}{3}} x_i \tag{11}$$

This scaling gives us

$$\begin{aligned}
\frac{\partial \vec{S}}{\partial t'} = & -\vec{S} \times \left( -\Delta \vec{S} - \frac{1}{2} (-r' + 2r'_0 - r'_k - |r' - r'_k| + |r' - 2r'_0 + r'_k + |r' - r'_k|) \vec{S} \cdot \hat{z} \hat{z} \right. \\
& \left. - (|r' - r'_0| + r' - r'_0) \vec{S} \cdot \hat{r} \hat{r} - \vec{B}' \right) \\
& + \lambda \vec{S} \times \left( \vec{S} \times \left( -\Delta \vec{S} - \frac{1}{2} (-r' + 2r'_0 - r'_k - |r' - r'_k| + |r' - 2r'_0 + r'_k + |r' - r'_k|) \vec{S} \cdot \hat{z} \hat{z} \right. \right. \\
& \left. \left. - (|r' - r'_0| + r' - r'_0) \vec{S} \cdot \hat{r} \hat{r} - \vec{B}' \right) \right)
\end{aligned} \tag{12}$$

Since the damping term does not contribute to static properties of the system, the critical value of external magnetic field intensity depends directly on the proportionality coefficient between  $B$  and  $B'$ . As such we have already obtained qualitative dependence of critical field value on material parameters. Numerical analysis of the system presented in this paper will also provide dependence of critical field value on the geometrical parameters of the wire.

## 2.3 Numerical scheme

For the purpose of numeric calculations it is additionally assumed that the investigated solutions have cylindrical symmetry, so only dependence on radius is considered. It is also considered that  $r'_0 \gg r'_0 - r'_k$ , which warrants the use of Cartesian coordinates instead of an explicitly cylindrical system. This approximation also makes  $r'_k - r'_0$  the only relevant geometrical parameter of the microwire.

Given equations of motion are solved numerically using finite difference method. For the purpose of determining the critical field for magnetisation switching process, we check the stability of the static solution for  $B = 0$  under magnetic field opposite to the bulk magnetisation direction under an additional simplification of an overdamped system

$$\begin{aligned} \frac{\partial \vec{S}}{\partial t'} = & \lambda \vec{S} \times \left( \vec{S} \times \left( -\Delta \vec{S} - \frac{1}{2} (-r' + 2r'_0 - r'_k - |r' - r'_k| + |r' - 2r'_0 + r'_k + |r' - r'_k|) \vec{S} \cdot \hat{z} \hat{z} \right. \right. \\ & \left. \left. - (|r' - r'_0| + r' - r'_0) \vec{S} \cdot \hat{r} \hat{r} - \vec{B}' \right) \right) \end{aligned} \quad (13)$$

which essentially is akin to the steepest descent method of obtaining an equilibrium of a system. For subcritical values of  $B$  the solution deforms, yet stabilises with magnetisation never reaching the same direction as the external magnetic field opposite to the initial magnetisation of the bulk. For overcritical fields the new domain aligned with the external field forms at the point, where anisotropy is the lowest, and expands unconstrained toward the centre of the wire (see figure 1). Thus the numerical criterion taken was whether the magnetic moment reaches the external magnetic field direction or not with a set angle margin of  $0.0008 \text{ rad}$  (other values were also tested). For a given value of  $r'_k - r'_0$  a series of simulations was undertaken to estimate the nucleation field as the lowest field intensity for which the solution is unstable, which was obtained by taking an interval of  $B'$  values, for which one extreme was subcritical and the other overcritical and iterated bisection thereof until the width of the interval is no larger than a stated precision goals, which was picked as 0.001 in our chosen units defined by the scaling procedure. It is important to note that close to the critical value of  $B'$  reaching the point, where divergent solutions can be distinguished, takes progressively more simulation time. For this reason simulations were repeated with varying artificial time in order to make sure we do not overestimate the nucleation field within chosen precision goal.

## 3 Results

### 3.1 Analytic correlations

If we consider the scaled equation in the Cartesian approximation, then the critical field value  $B'_c$  can only depend on  $r'_k - r'_0$ . Thus we have

$$B_c = \frac{(d^2 J)^{\frac{1}{3}}}{g \mu_B} B'_c(r'_k - r'_0) \quad (14)$$

with  $r'_k - r'_0$  still dependent on  $d$  and  $J$  due to scaling - this will be considered later in the context of numerical results for  $B'_c(r'_k - r'_0)$ . It is most interesting that the key parameter is the rate of anisotropy change and not the maximal value thereof in the bulk. This means that due to the connection between anisotropy and stress inside the wire [4] one can manipulate the nucleation field by manipulating the stress gradient inside the wire.

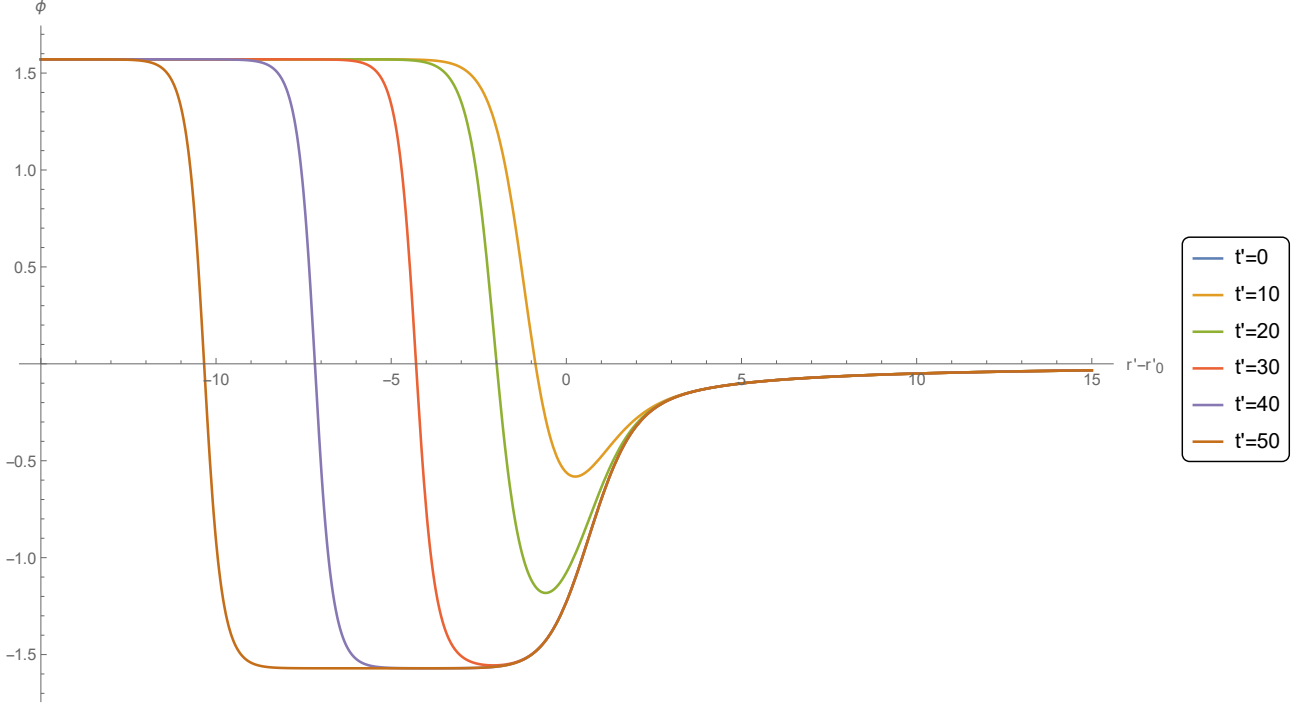


Figure 1: Magnetisation angle as a function of radius at various time points

### 3.2 Numerical analysis

After initial simulations for varying values of  $r'_k - r'_0$  we established that a relevant interval for investigation is  $r'_k - r'_0 \in < -3.0, -0.2 >$ . The value of nucleation field remains constant below  $r'_k - r'_0 = -3.0$  and for  $r'_k - r'_0 > -0.2$  the system exhibit anomalous behaviour, since the axial anisotropy in the bulk becomes insignificantly small. For the chosen interval simulations were repeated with varying simulation time. Below (figure 2) we present obtained data for  $r'_k - r'_0 = -3.0$  and  $r'_k - r'_0 = -0.2$  as examples. The steps of values visible one the plot come

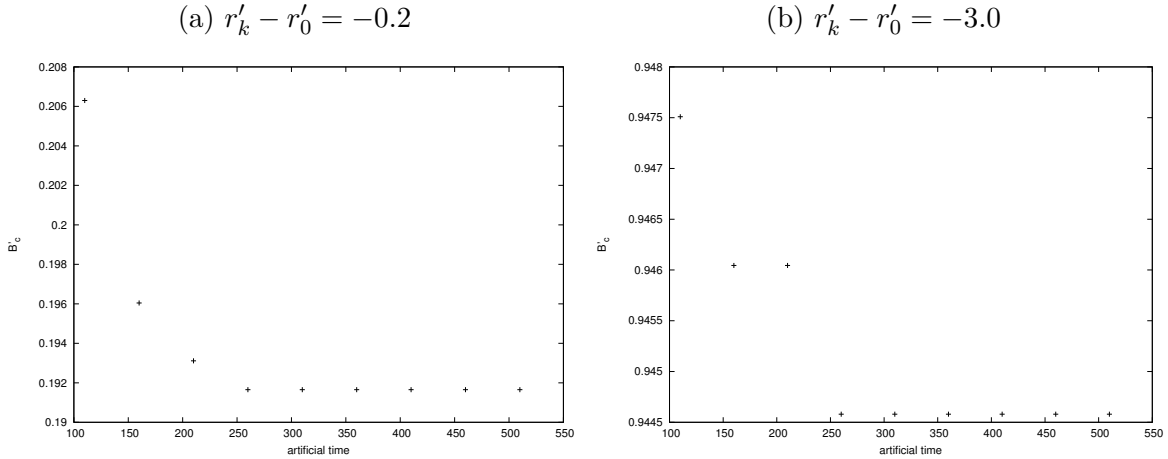


Figure 2: Nucleation field vs. simulation time

from the chosen precision goal of nucleation field. As can be seen, setting the simulation time to a value higher then 300 gives us proper results with chosen precision goal. With that in

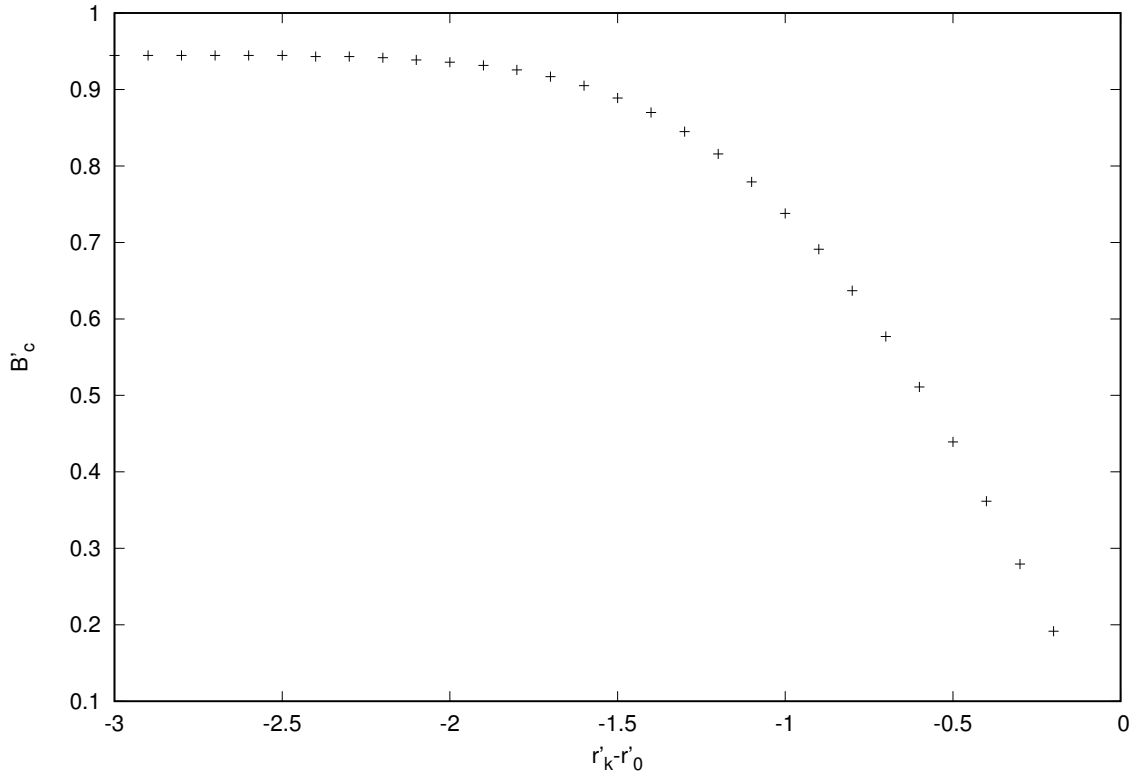


Figure 3: Nucleation field vs. intermediate layer width

mind we took the data for artificial time value of 510 and obtained the correlation between nucleation field value and  $r'_k - r'_0$  (figure 3)

Additionally we investigated, how the choice of the angle margin impacts the results. For the simulation time value of 510 and a margin of  $6 \cdot 10^{-6} \text{ rad}$  the simulations were repeated and compared with previously obtained (figure 4).

For higher values of  $|r'_k - r'_0|$  both simulation series give the same results up to the chosen precision goal. For the narrower angle margin the anomalous behaviour starts significantly sooner than for the wider one. This observation indicates that the anomaly is artificial. The explanation we found is that for thin intermediate layer the magnetisation is switched very early, but will not align exactly with the external magnetic field due to the influence of the radially oriented domain and lack of strong axial anisotropy in the bulk. This hypothesis was confirmed by the obtained magnetisation profiles for those cases.

## 4 Discussion

The most important result is that there is a specific limit on  $B'_c$  value for high  $|r'_k - r'_0|$ , which means that the nucleation field is not dependent on the maximum anisotropy in the bulk and the anisotropy change rate is the deciding factor. This may seem to contradict results of [14, 4], where only the global qualities of the microwire are taken into account. Those works however focused on switching field and on global average qualities of the microwires, while our result is the local nucleation field and focuses on where a new domain is actually created.

It is also important to stress that the simulations clearly show, how the domain nucleation

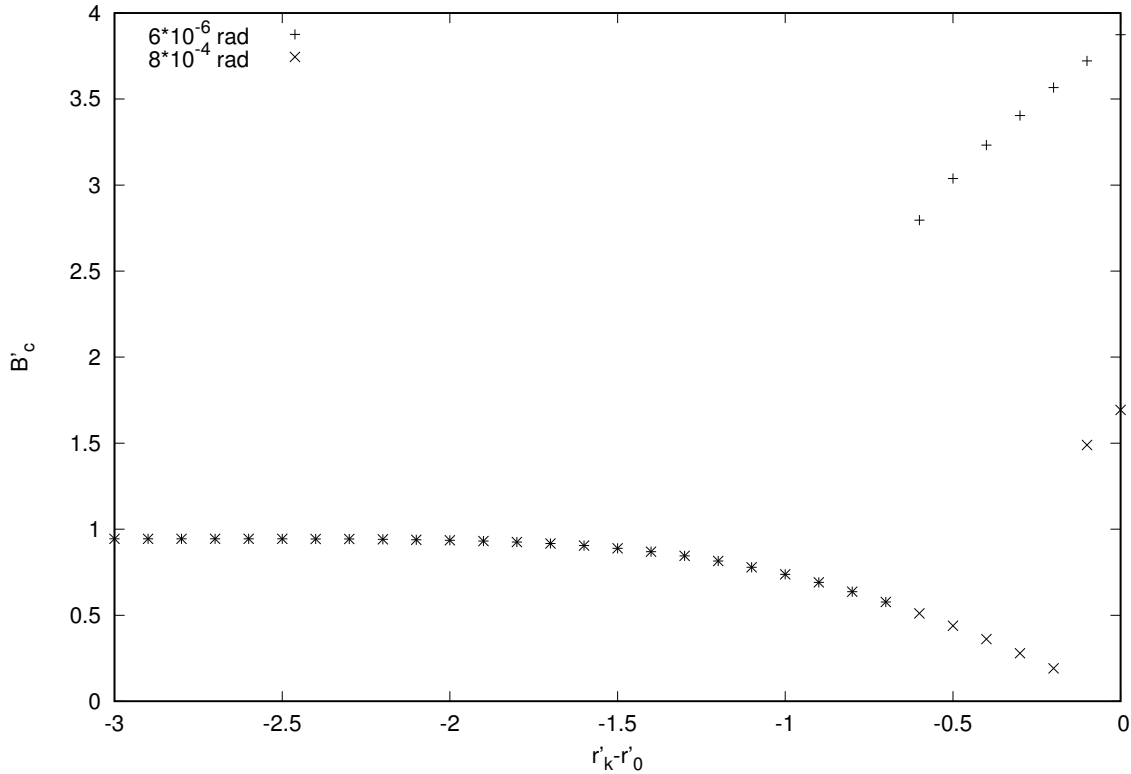


Figure 4: Nucleation field vs. intermediate layer width for different angle margins

process starts in the intermediate layer and even for subcritical fields the change in magnetisation is clearly visible. This may be the key to understanding the domain wall propagation process in bistable microwires - both for the speed and the elongated shape. The general idea for further research would be to consider a movement of a domain wall not along the wire but rather view it as a spreading process of domain nucleation in the intermediate layer and a domain wall movement from the surface toward the centre of the wire.

Our estimation however does not account for demagnetising field directly and as such will not be suitable for short microwires [14]. Cartesian coordinates approximation also makes the results invalid for particularly thin wires. The decline in nucleation field value, when the width of the intermediate layer becomes smaller, is directly connected to the value of anisotropy in the bulk though, but it won't necessarily be observed in real microwires, since if we take parameter estimations from [4] and assume the exchange constant for pure iron to be valid for the  $Fe_{77.5}Si_{7.5}B_{15}$  compound, then  $r'_k - r'_0 \approx -5$  which is far in the stable region of the graph. It is difficult to take it as a general statement though, since there is little data on anisotropy distribution within glass-coated microwires. While the method of calculating internal stress is well understood [4], few experimental papers contain information on wire production conditions, which is necessary for calculating anisotropy distribution. Dedicated experiments are planned for further work.

## 5 Conclusions

Domain nucleation process clearly starts in that area of the wire, where the anisotropy changes its character and there is a clear analytic relation between the nucleation field value and internal parameters of the wire.

Nucleation field depends on the anisotropy change rate ( $\frac{\partial D_1}{\partial r}$ ), which can be controlled by production conditions, annealing and other processing procedures.

Study of domain nucleation process in microwires gives valuable insight into domain wall propagation and gives a direction for further research on the subject.

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